Day 6: GEOMETRIC SERIES

1 + 4 + 16 + 64 + 256

- The terms of a geometric sequence expressed as a sum

SUM OF A GEOMETRIC SERIES

$$S_n = \frac{t_1(r^n - 1)}{r - 1}, \quad r \neq 1 \quad *$$

need to know "n"

OR $$S_n = \frac{rt_n - t_1}{r - 1}, \quad r \neq 1$$

need to know t_n

t_1 = first term
t_n = n^{th} term
r = common ratio
S_n = sum of the first n

*see p. 48-49 for developing formula

EX. 1 Determine the sum of the first 10 terms of each geometric series

$n = 10$ r

a) 4 + 12 + 36 + ...

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_{10} = \frac{4(3^{10} - 1)}{3 - 1}$$

$$S_{10} = 118{,}096$$

b) $t_1 = 5$ and r = ½

$$S_n = \frac{t_1(r^n - 1)}{r - 1}$$

$$S_n = \frac{5(\frac{1}{2}^{10} - 1)}{\frac{1}{2} - 1}$$

$$S_n = 9.99$$

EX. 2 Determine the sum of each geometric series.

Method 1: Determine Number of Terms

a) $\frac{1}{27} + \frac{1}{9} + \frac{1}{3} + \ldots + 729$ $\leftarrow t_n$ $n = ?$

$$S_n = \frac{rt_n - t_1}{r - 1}$$

$$S_n = \frac{3(729) - \frac{1}{27}}{3 - 1}$$

$$S_n = 1093.5$$

$$r = \frac{T_2}{T_1}$$

$$r = \frac{\frac{1}{9}}{\frac{1}{27}}$$

$$r = \frac{1}{9} \div \frac{1}{27}$$

$$r = \frac{1}{9} \times \frac{27}{1}$$

$$r = \frac{27}{9} \text{ or } 3$$

Method 2: Use second Formula $S_n = \frac{rt_n - t_1}{r - 1}$

b) 4 – 16 + 64 - ... - 65536

TRICKIER $\boxed{r = -4}$

$n = ?$

$$S_n = \frac{rt_n - t_1}{r - 1}$$

$$S_n = \frac{(-4)(-65536) - (4)}{-4 - 1}$$

$$S_n = -52428$$

EX. 3 Your parents have come up with a reward system for the summer holidays if you keep your room clean! The deal is you are given a penny on the first day of summer vacation, two pennies on the second day, four pennies on the third day, eight pennies on the fourth day and so on… doubling for each succeeding day.

Summer vacation was 64 days long

a) Write the first seven terms of this geometric sequence.

1¢, 2¢, 4¢, 8¢, 16¢, 32, 64

$1 + 2 + 4 + 8 + 16 + 32 + 64 = 128$¢

$1.28

b) Write an expression that shows the sum of all the money earned in the first week.

c) How much money do your parents give you on the 64th day?

$t_{64} = t_1 r^{n-1}$

$t_{64} = 0.01(2)^{64-1}$

$t_{64} = 9.2E16$

9.2×10^{16}

$92,000,000,000,000,000.00

d) Determine how much money you would earn if you kept your room clean all summer.

$$S_n = \frac{t_1(r^n - 1)}{r-1}$$

$$S_n = \frac{0.01(2^{64} - 1)}{2-1}$$

$S_n = 1.84 \times 10^{17}$

1840 000 000 000 000 00

Worksheet: Geometric Series

1. Find S_7 for each series:

   a) $5 - 10 + 20 - 40 + \ldots$ b) $12 + 6 + 3 + 1.5 + \ldots$ **S_7= 215**

   S_7= 23.8125

2. Find the sum of the given series: $1 + 5 + 25 + \ldots + 3125$ **S_n= 3906**

3. A doctor prescribes 200mg of medication on the first day of treatment. The dosage is halved on each successive day for 1 week. To the nearest milligram, what is the total amount of medication administered?

   S_n= 397 mg

4. Find S_n for a series with $t_n = 5(2)^{n-1}$. Careful!!! **S_n= $5(2^n-1)$**

BONUS: The second term of a geometric series is 15 and the sum of the first 3 terms is 93. Find the first 3 terms of the series. Show your work on the back.