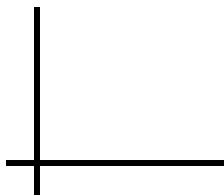




os

INVESTIGATE:

A) TRIGONOMETRIC RATIOS OF ANY ANGLE $0^\circ \leq \theta \leq 360^\circ$



- θ is any angle in standard position
- $P(x, y)$ is any point on its terminal arm, at a distance "r" from the origin
- Reference triangle is created by drawing a line perpendicular to the x-axis through point $P(x, y)$

1) Use the Pythagorean Theorem to show the relationship between x, y and r . Solve for r .

$r = \underline{\hspace{2cm}}$

2) Use primary trigonometric ratios to show the relationship between θ, x, y and r .

a) $\sin \theta = \underline{\hspace{2cm}}$ b) $\cos \theta = \underline{\hspace{2cm}}$ c) $\tan \theta = \underline{\hspace{2cm}}$

B) SIGNS OF TRIGONOMETRIC RATIOS IN EACH QUADRANT

- Plot the given point $P(x, y)$. Sketch reference triangle.
- Label the point, sides of triangle, θ , and θ_R . Determine primary trig ratios.

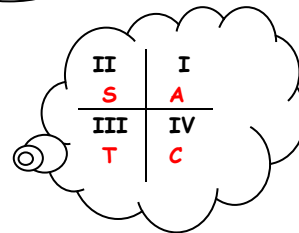
<p>Quadrant II: $90^\circ < \theta < 180^\circ$</p> <p style="text-align: center;">Plot $(-4, 3)$</p> <p>$\sin \theta = \underline{\hspace{2cm}}$</p> <p>$\cos \theta = \underline{\hspace{2cm}}$</p> <p>$\tan \theta = \underline{\hspace{2cm}}$</p>	<p>Quadrant I: $0^\circ < \theta < 90^\circ$</p> <p style="text-align: center;">Plot $(4, 3)$</p> <p>$\sin \theta = \underline{\hspace{2cm}}$</p> <p>$\cos \theta = \underline{\hspace{2cm}}$</p> <p>$\tan \theta = \underline{\hspace{2cm}}$</p>
<p>Quadrant III: $180^\circ < \theta < 270^\circ$</p> <p style="text-align: center;">Plot $(-4, -3)$</p> <p>$\sin \theta = \underline{\hspace{2cm}}$</p> <p>$\cos \theta = \underline{\hspace{2cm}}$</p> <p>$\tan \theta = \underline{\hspace{2cm}}$</p>	<p>Quadrant IV: $270^\circ < \theta < 360^\circ$</p> <p style="text-align: center;">Plot $(4, -3)$</p> <p>$\sin \theta = \underline{\hspace{2cm}}$</p> <p>$\cos \theta = \underline{\hspace{2cm}}$</p> <p>$\tan \theta = \underline{\hspace{2cm}}$</p>

CONCLUSION: "ALL STUDENTS TAKE CALCULUS"

OR "CAST"

- All trig functions are positive in
- Sine is also positive in
- Tangent is also positive in
- Cosine is also positive in

quadrant: _____
 quadrant: _____
 quadrant: _____
 quadrant: _____

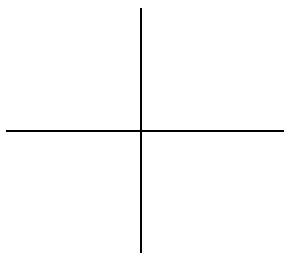


WRITE TRIGONOMETRIC RATIOS FOR ANGLES IN ANY QUADRANT

- ① Sketch and label the reference triangle → point, sides of triangle, θ , and θ_R
- ② Determine the distance, r , from the $P(x, y)$ to the origin → Pythagorean Theorem
- ③ Write out the primary trigonometric ratios → $\sin \theta$, $\cos \theta$ and $\tan \theta$
- ④ Check answer makes sense → quadrant signs ALL STUDENTS TAKE CALCULUS?

EX. 1 The point $P(-8, 15)$ lies on the terminal arm of an angle, θ , in standard position. Determine the exact trigonometric ratios for $\sin \theta$, $\cos \theta$ and $\tan \theta$.

① Reference Triangle ② $r =$ _____



③ Trig Ratios

$\sin \theta =$ _____
 $\cos \theta =$ _____
 $\tan \theta =$ _____

④ ASTC?

Quadrant: _____

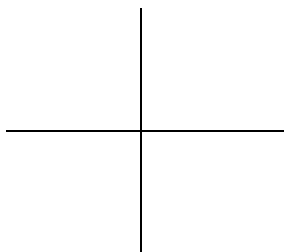
See Example 1 & Your Turn

p. 91

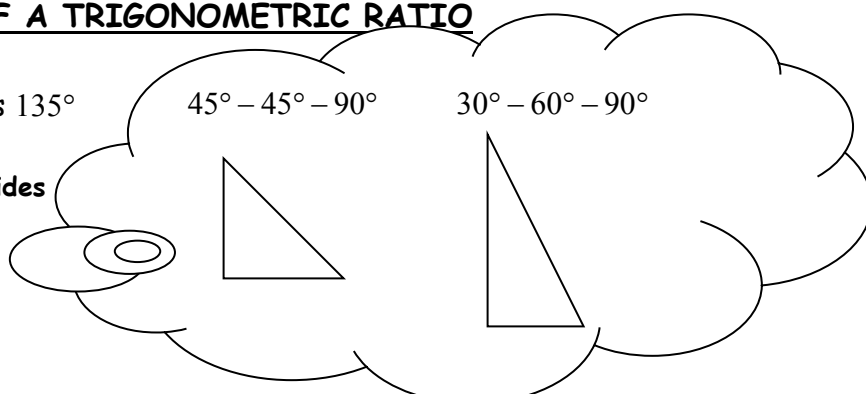
DETERMINE THE EXACT VALUE OF A TRIGONOMETRIC RATIO

EX. 2 Determine the exact value of $\cos 135^\circ$

① Reference Triangle ② label sides



$\cos 135^\circ =$ _____



③ Trig Ratios

$\cos \theta = \cos \theta_R$

$\cos 135^\circ = \cos$ _____

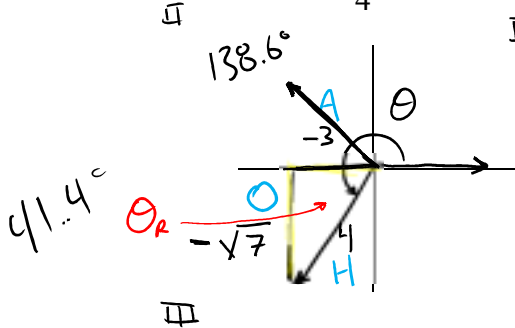
④ ASTC?

Quadrant: _____

DETERMINE TRIGONOMETRIC RATIOS

EX. 3 Suppose θ is an angle in standard position with terminal arm in quadrant III, and

$\cos \theta = -\frac{3}{4}$. What are the exact values of $\sin \theta$ and $\tan \theta$?



$$\cos = \frac{x}{r}$$

$$\cos \theta = \frac{-3}{4}$$

$$a^2 + b^2 = c^2$$

$$a^2 + (-3)^2 = 4^2$$

$$a^2 + 9 = 16$$

$$a^2 = 7$$

$$a = \sqrt{7}$$

What is $\theta = \underline{\quad}$?

$\tan \theta = \frac{\sqrt{7}}{3}$ $\sin \theta = \frac{-\sqrt{7}}{4}$

$\theta = 180 + 41.4 = 221.4^\circ$

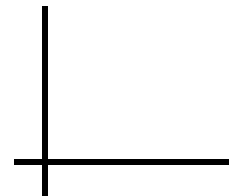
-180.0

$\theta = 138.6$

QUADRANTAL ANGLES: $0^\circ, 90^\circ, 180^\circ, 270^\circ,$ and 360°

- An angle in standard position whose terminal arm lies on an axis (x or y)
- We can't draw a reference triangle, but we can think about the relationship between x, y, r and θ .

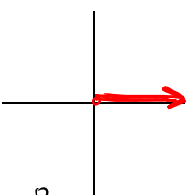
$$\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x}$$



DETERMINE TRIGONOMETRIC RATIOS OF QUADRANTAL ANGLES

EX. 4 Determine the values of $\sin \theta, \cos \theta,$ and $\tan \theta$ when the terminal arm of quadrantal angle θ coincides with the :

a) positive x-axis
 $\theta = 0^\circ$

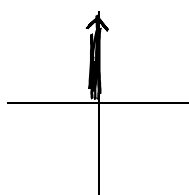


$$\cos 0^\circ = 1$$

$$\sin 0^\circ = 0$$

$$\tan 0^\circ = 0$$

b) positive y-axis
 $\theta = 90^\circ$

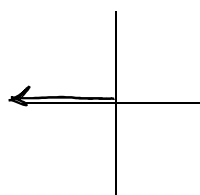


$$\cos 90^\circ = 0$$

$$\sin 90^\circ = 1$$

$$\tan 90^\circ = \phi$$

c) negative x-axis
 $\theta = 180^\circ$

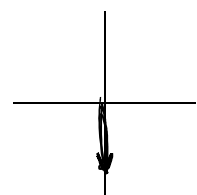


$$\cos 180^\circ = -1$$

$$\sin 180^\circ = 0$$

$$\tan 180^\circ = 0$$

d) negative y-axis
 $\theta = 270^\circ$



$$\cos 270^\circ = 0$$

$$\sin 270^\circ = -1$$

$$\tan 270^\circ = \phi$$

SOLVING FOR ANGLES GIVEN THEIR SINE, COSINE OR TANGENT

- ① Look at the sign (+ or -) of the ratio and use "ASTC"
 - Which quadrant(s) will the solutions be in
- ② Ignore the sign of the ratio and determine the reference angle
 - Using your calculator (2nd sin, cos or tan)
 - using a special triangle
- ③ Sketch the reference angle in the appropriate quadrants
 - Use diagram to determine the measure of the angles in standard position

SOLVE FOR AN ANGLE GIVEN ITS EXACT SINE, COSINE, OR TANGENT VALUE

EX. 5 Solve for θ

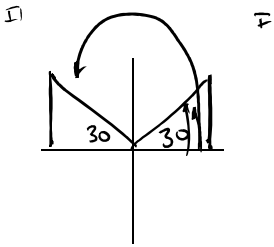
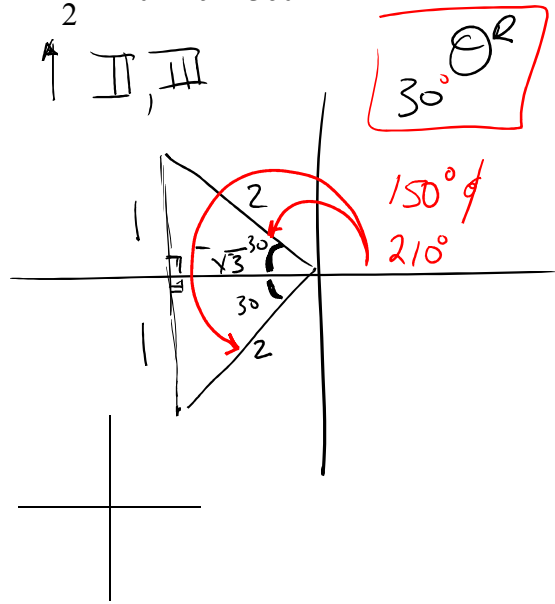
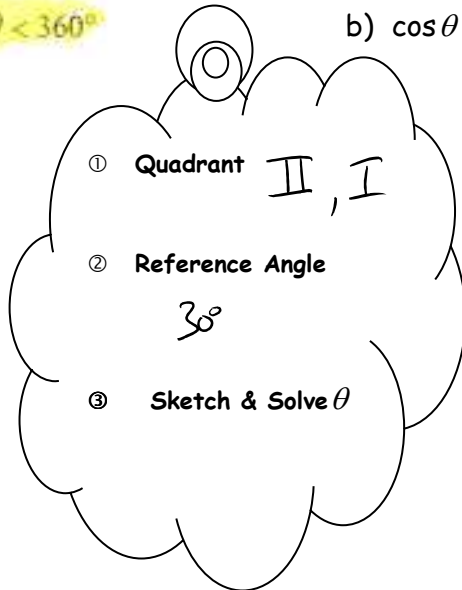
a) $\sin \theta = 0.5$

More than 1
clue ↓ answer!

$0^\circ \leq \theta < 360^\circ$

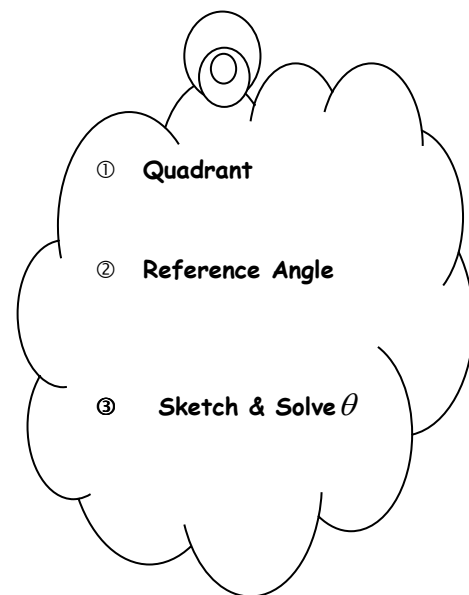
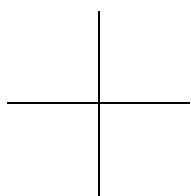
b) $\cos \theta = -\frac{\sqrt{3}}{2}$ $0^\circ \leq \theta < 360^\circ$

"+" ↑
 30° & 150°



SOLVE FOR AN ANGLE GIVEN ITS APPROXIMATE SINE, COSINE, OR TANGENT VALUE

EX. 6 Given $\cos \theta = -0.6753$, where $0^\circ \leq \theta < 360^\circ$, determine the measure of θ , to the nearest tenth of a degree.



See Example 6 & Your Turn

p. 95

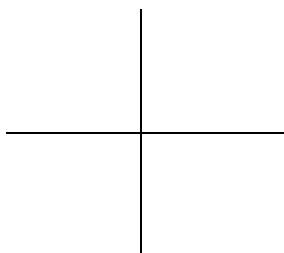
PC 11 2.2 W/S TRIGONOMETRIC RATIOS OF ANY ANGLE



- Show all work and attach to textbook work for this section

1. The point $P(5,-2)$ lies on the terminal arm of an angle, θ , in standard position.
Determine the exact trigonometric ratios for $\sin \theta$, $\cos \theta$ and $\tan \theta$. (See EX 1)

① Reference Triangle ② $r =$ _____



③ Trig Ratios

$\sin \theta =$ _____

$\cos \theta =$ _____

$\tan \theta =$ _____

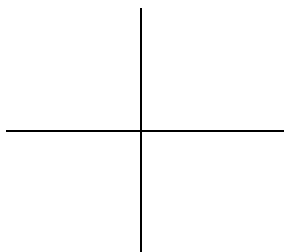
④ **ASTC?**

Quadrant: _____

2. Determine the exact value of (See EX 2)

a) $\sin 240^\circ$

① Reference Triangle ② special triangle



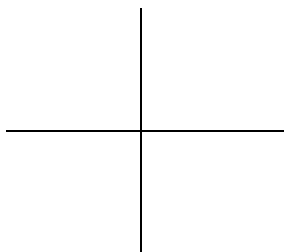
③ Trig Ratios

④ **ASTC?**

Quadrant: _____

b) $\tan 210^\circ$

① Reference Triangle ② special triangle



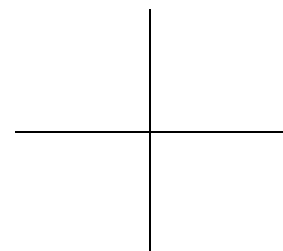
③ Trig Ratios

④ **ASTC?**

Quadrant: _____

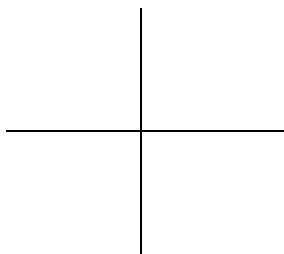
3 a) Suppose θ is an angle in standard position with terminal arm in quadrant II, and $\sin \theta = \frac{4}{5}$.

What are the exact values of $\cos \theta$ and $\tan \theta$? (See EX 3)

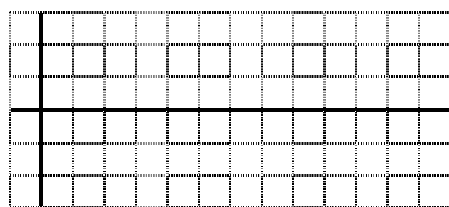


b) Suppose θ is an angle in standard position with terminal arm in quadrant III, and $\tan \theta = \frac{3}{7}$.

What are the exact values of $\cos \theta$ and $\sin \theta$?

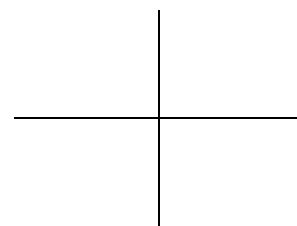
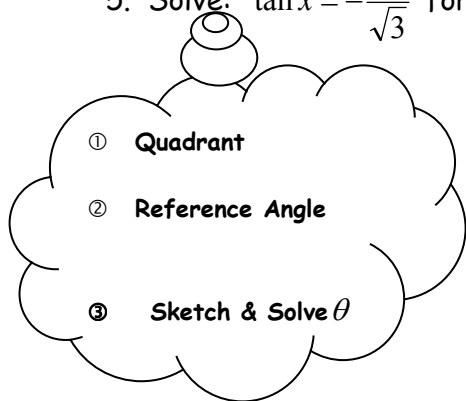


4. Plot the following table of values on a graph where θ° values are on the x-axis and $\sin \theta$ values are on the y-axis



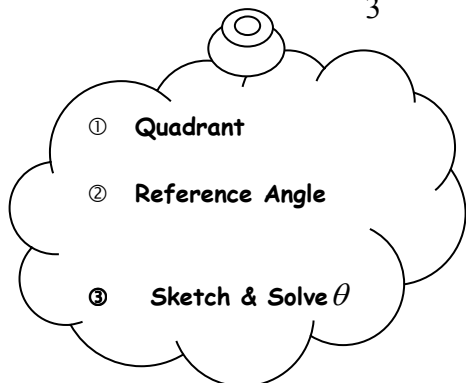
θ°	$\sin \theta$
0	0
30	0.5
90	1
150	0.5
180	0
210	-0.5
270	-1
330	-0.5
360	0

5. Solve: $\tan x = -\frac{1}{\sqrt{3}}$ for $0^\circ \leq \theta < 360^\circ$. (exact solutions) (See EX 5)

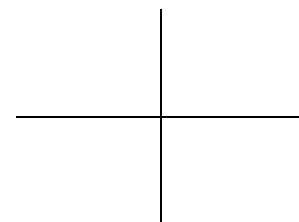


X = _____

6. Solve: $\sin x = -\frac{1}{3}$ for $0^\circ \leq \theta < 360^\circ$. (2 decimal places) (See EX 6)



AY 3 HWORK TONIGHT



x = _____