

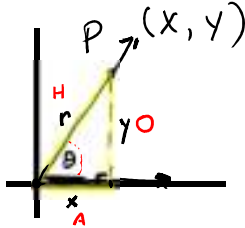
* DAY 3 NOTES *

PC 11 2.2 TRIGONOMETRIC RATIOS OF ANY ANGLE



INVESTIGATE:

A) TRIGONOMETRIC RATIOS OF ANY ANGLE $0^\circ \leq \theta \leq 360^\circ$



- θ is any angle in standard position
- $P(x, y)$ is any point on its terminal arm, at a distance "r" from the origin
- Reference triangle is created by drawing a line perpendicular to the x-axis through point $P(x, y)$

1) Use the Pythagorean Theorem to show the relationship between x, y and r. Solve for r.

$$a^2 + b^2 = c^2$$

$$x^2 + y^2 = r^2$$

$$\sqrt{x^2 + y^2} = r \quad \leftarrow \text{please remember this}$$

r = _____

2) Use primary trigonometric ratios to show the relationship between θ , x, y and r.

a) $\sin \theta = \frac{O}{H} \text{ or } \frac{y}{r}$ b) $\cos \theta = \frac{A}{r}$ c) $\tan \theta = \frac{y}{x}$

B) SIGNS OF TRIGONOMETRIC RATIOS IN EACH QUADRANT

- Plot the given point $P(x, y)$. Sketch reference triangle.
- Label the point, sides of triangle, θ , and θ_R . Determine primary trig ratios.

<p>Quadrant II: $90^\circ < \theta < 180^\circ$</p> <p>$r = \sqrt{x^2 + y^2}$ $r = \sqrt{9 + 16}$ $r = \sqrt{25}$ $r = 5$</p> <p>Plot (-4, 3)</p> <p>$\sin \theta = \frac{3}{5}$</p> <p>$\cos \theta = \frac{-4}{5}$</p> <p>$\tan \theta = \frac{3}{-4}$</p>	<p>Quadrant I: $0^\circ < \theta < 90^\circ$</p> <p>Plot (4, 3)</p> <p>$\sin \theta = \frac{3}{5}$</p> <p>$\cos \theta = \frac{4}{5}$</p> <p>$\tan \theta = \frac{3}{4}$</p>
<p>Quadrant III: $180^\circ < \theta < 270^\circ$</p> <p>Plot (-4, -3)</p> <p>$\sin \theta = \frac{-3}{5}$</p> <p>$\cos \theta = \frac{-4}{5}$</p> <p>$\tan \theta = \frac{-3}{-4} \rightarrow \frac{3}{4}$</p>	<p>Quadrant IV: $270^\circ < \theta < 360^\circ$</p> <p>Plot (4, -3)</p> <p>$\sin \theta = \frac{-3}{5}$</p> <p>$\cos \theta = \frac{4}{5}$</p> <p>$\tan \theta = \frac{-3}{4}$</p>

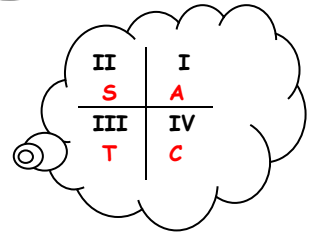
Mnemonic

OR "CAST"

CONCLUSION: "ALL STUDENTS TAKE CALCULUS"

- **All** trig functions are positive in
- **Sine** is also positive in
- **Tangent** is also positive in
- **Cosine** is also positive in

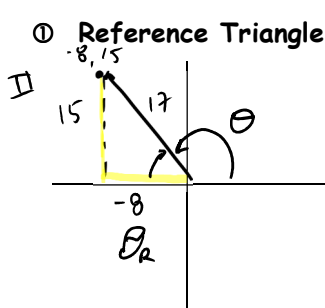
quadrant: I
 quadrant: II
 quadrant: III
 quadrant: IV



WRITE TRIGONOMETRIC RATIOS FOR ANGLES IN ANY QUADRANT

- ① Sketch and label the reference triangle → point, sides of triangle, θ , and θ_R
- ② Determine the distance, r , from the $P(x, y)$ to the origin → Pythagorean Theorem
- ③ Write out the primary trigonometric ratios → $\sin \theta$, $\cos \theta$ and $\tan \theta$
- ④ Check answer makes sense → quadrant signs **ALL STUDENTS TAKE CALCULUS?**

EX. 1 The point $P(-8, 15)$ lies on the terminal arm of an angle, θ , in standard position. Determine the exact trigonometric ratios for $\sin \theta$, $\cos \theta$ and $\tan \theta$.



② $r = 17$
 $r = \sqrt{(15)^2 + (-8)^2}$
 $r = 17$

③ Trig Ratios

$\sin \theta = \frac{15}{17}$
 $\cos \theta = \frac{-8}{17}$
 $\tan \theta = \frac{15}{-8}$

④ **ASTC?**

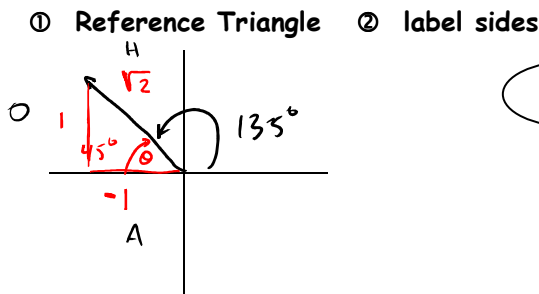
Quadrant: II

See Example 1 & Your Turn

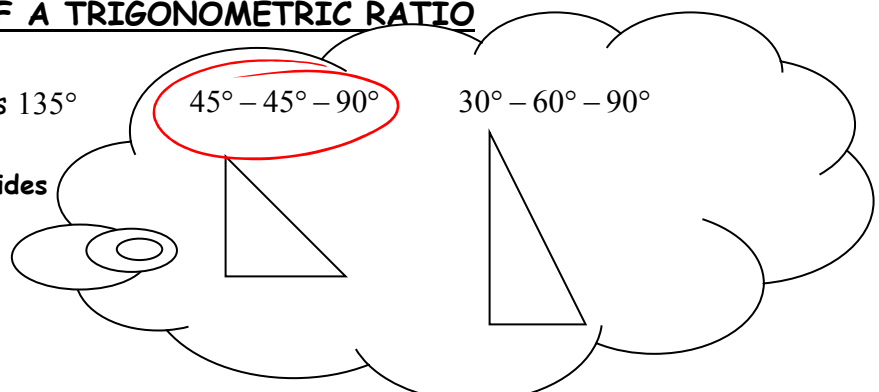
p. 91

DETERMINE THE EXACT VALUE OF A TRIGONOMETRIC RATIO

EX. 2 Determine the exact value of $\cos 135^\circ$



$\cos 135^\circ = \frac{-1}{\sqrt{2}}$



③ Trig Ratios

$\cos \theta = \cos \theta_R$

④ **ASTC?**

Quadrant: II

$\cos 135^\circ = \cos 45^\circ$

$x = \underline{\hspace{2cm}}$