

"Completing the Square" Day 6 Notes

Note Title

19/10/2017

I. Converting standard form to vertex form
 $y = ax^2 + bx + c \leftrightarrow y = a(x-p)^2 + q$

II Easy

e.g.1 $y = x^2 + 6x + 5$

$$y = (x^2 + 6x) + 5$$

$$(x+3)(x+3)$$

$$y = (x^2 + 6x + 9 - 9) + 5$$

$$y = (x+3)^2 - 9 + 5$$

$$y = (x+3)^2 - 4$$

① Bracket 1st 2 terms

② Take $\frac{1}{2}b$ &

square it!
add it & sub it

③ Box the trinomial

$$V(-3, -4)$$

parabola up ↑



e.g.2 $y = x^2 - 4x + 7$
 $y = (x^2 - 4x) + 7$

$$y = (x^2 - 4x + 4 - 4) + 7$$

$$y = (x-2)^2 - 4 + 7$$

$$y = (x-2)^2 + 3$$

$$V(2, 3) \uparrow$$

III Medium

$a \neq 1$ i.e. coefficient in front

e.g.1 $y = 2x^2 - 20x$
 $y = 2(x^2 - 10x)$

extra step: Factor out the "a" value first.

$$y = 2(x^2 - 10x + 25 - 25)$$

$$y = 2(x-5)^2 - 25$$

$$y = 2(x-5)^2 - 50 \quad V(5, -50)$$

e.g. 2 $y = -3x^2 - 18x - 24$

$$y = (-3x^2 - 18x) - 24$$

$$y = -3(x^2 + 6x) - 24$$

$$y = -3(\boxed{x^2 + 6x + 9} - 9) - 24$$

$$y = \textcircled{-3} [(x+3)^2 - 9] - 24$$

$$y = -3(x+3)^2 + 27 - 24$$

$$y = -3(x+3)^2 + 3$$

$$V(-3, 3)$$

opens ↓

VERTICALLY
EXPANDED



TRY

$$y = -4x^2 + 24x - 35$$

$$y = (-4x^2 + 24x) - 35$$

$$y = -4(x^2 - 6x) - 35$$

$$y = -4(\boxed{x^2 - 6x + 9} - 9) - 35$$

$$y = -4[(x-3)^2 - 9] - 35$$

$$y = -4(x-3)^2 + 36 - 35$$

$$y = -4(x-3)^2 + 1$$

III Hard - FRACTIONS ARE FUN 😊

$$y = 2x^2 - 2x + 1$$

$$y = (2x^2 - 2x) + 1$$

$$y = 2(x^2 - 1x) + 1$$

$$y = 2(\boxed{x^2 - x + \frac{1}{4}} - \frac{1}{4}) + 1$$

$$y = 2\left[\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}\right] + 1$$

$$\frac{1}{2}x - 1 = -\frac{1}{2}$$

$$\left(-\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\frac{2x - 1}{1} = \frac{-2}{4}$$

$$= -\frac{1}{2}$$

$$y = 2\left(x - \frac{1}{2}\right)^2 - \frac{1}{2} + 1$$

$$y = 2\left(x - \frac{1}{2}\right)^2 + \frac{1}{2}$$

e.g. $y = 4x^2 - 28x - 23$

FRACTION

$$y = (4x^2 - 28x) - 23$$

$$y = 4(x^2 - 7x) - 23$$

$\left(\frac{7}{2}\right)^2$

$$y = 4\left(x^2 - 7x + \frac{49}{4} - \frac{49}{4}\right) - 23$$

$$y = 4\left[\left(x - \frac{7}{2}\right)^2 - \frac{49}{4}\right] - 23$$

$$y = 4\left(x - \frac{7}{2}\right)^2 - 49 - 23$$

$$y = 4\left(x - \frac{7}{2}\right)^2 - 72$$

DECIMAL

$$y = (4x^2 - 28x) - 23$$

$$y = 4(x^2 - 7x) - 23$$

$$y = 4\left(x^2 - 7x + 12.25 - 12.25\right) - 23$$

$$y = 4\left[\left(x - 3.5\right)^2 - 12.25\right] - 23$$

$$y = 4\left(x - 3.5\right)^2 - 49 - 23$$

$$y = 4\left(x - 3.5\right)^2 - 72$$

2 numbers that difference of 20

$$x, x + 20$$

$$x^2 + (x + 20)^2 = \text{min } y$$

their \rightarrow At the vertex

$$x^2 + x^2 + 40x + 400 = \text{min } y$$

$$2x^2 + 40x + 400 = \text{min } y \leftarrow \text{Standard}$$

\leftarrow vertex

"complete the square"