Linear Inequalities

January 12, 2018 9:40 AM

$< \quad > \quad \leq \quad \& \quad \geq$

9.1 – Linear Inequalities in Two Variables **Date:**

Key Ideas: $< \ > \ \leq \ \geq$ instead of "=" in equation

"ALSO REMEMBER TO FLIP THE SIGN WHEN × or ÷ by a negative

Example – Solve the inequality by graphing $3y - 2x \geq 6$. LINEAR

steps

1. Rearrange the inequality so it's in $mx + b$ form. Don't forget to flip the inequality if you multiply or divide by a negative number.

2. Decide whether to use a solid line or dotted line:

- If the inequality is ≤ or ≥, points on the line are included in the solution (due to the 'equals to' line under the sign), so we keep the line solid.
- If the inequality is < or >, points on the line are not included in the inequality, so we draw a dotted line.

3. Graph the line using slope and *y*-intercept. The line is called the **boundary**.

4. For y > mx + b or y ≥ mx + b, solutions to the inequality are all of the points **above** the line, so shade above. For y < mx + b or y ≤ mx + b, shade **below** the line. The shading represents the **solution region**: all of the points that satisfy the inequality.

5. CHECK: Pick a **test point** in the shaded region and test its coordinates in the inequality. If it satisfies the inequality, you've been successful. If it doesn't satisfy the inequality, you either shaded the incorrect region, or the boundary line has been graphed incorrectly.

slope y int

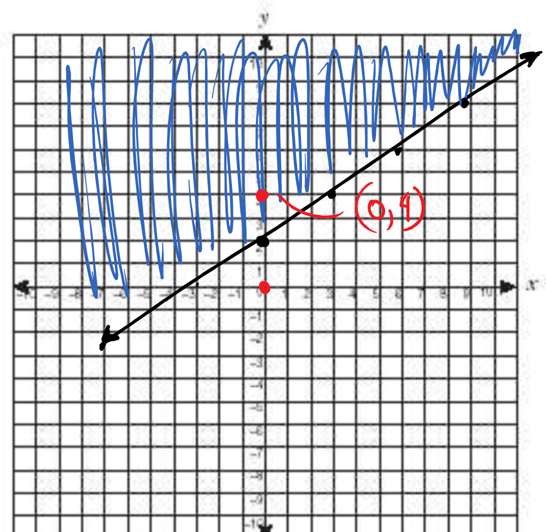
$y = mx + b$

$3y - 2x \geq 6$

$\frac{3y}{3} \geq \frac{2x + 6}{3}$

$\boxed{y \geq \frac{2}{3}x + 2}$

Slope $+\frac{2}{3}$ – $\frac{\text{rise}}{\text{run}}$ up 2 over 3



CHECK: WITH A TEST POINT

(0,0) $3y - 2x \geq 6$

$3(0) - 2(0) \geq 6$

$0 \geq 6$

FALSE!

(0,4) $3y - 2x \geq 6$

$3(4) - 2(0) \geq 6$

$12 \geq 6$

TRUE

CORRECT ☺

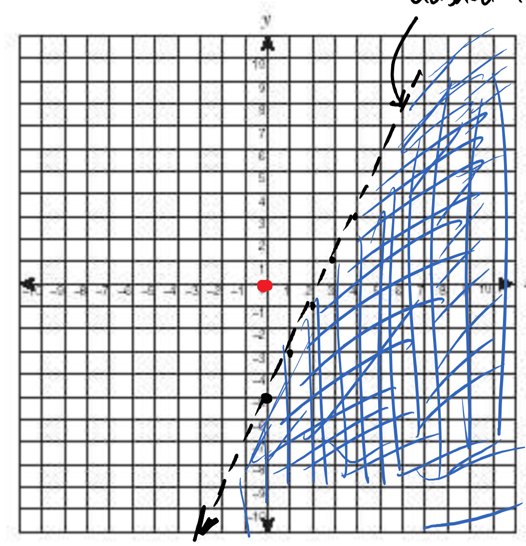
Example – Graph $4x - 2y > 10$. Determine if (1, 3) is part of the solution.

$$4x - 2y > 10$$
$$-2y > -4x + 10$$
$$y < \frac{-4x}{-2} + \frac{10}{-2}$$
$$y < 2x - 5$$

$\frac{2}{1}$ up over

3

FALSE



dashed line cuz $<$ not $\leq$ (solid line)

word problem

Example – Suppose you have received a gift card for a music downloading service. The card has a value of \$15. Individual songs (x) cost \$1 and a complete album (y) costs \$5. Write and graph a linear inequality to model the situation. Then describe all possible combinations of albums and songs that will cost \$15, and two possible combinations of albums and songs that will cost less than \$15.

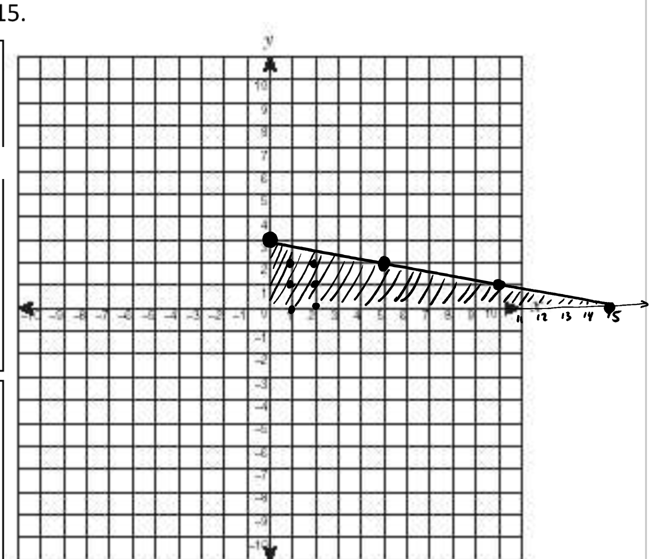
Let x = number of Songs

Let y = number of albums

inequality: $1x + 5y \leq 15$

rearranged:

$\rightarrow y \leq -\frac{1}{5}x + 3$



common sense restrictions:

no neg for x & y

all combinations that cost \$15:

15 x & 0 Y

two combinations less than \$15:

x^2 $> < \geq \leq$ x only so no "y" NO SHADING

9.2 – Quadratic Inequalities in One Variable **Date:**

Key Ideas: To solve using either GRAPHING OR ALGEBRA

Example – Solve $x^2 + 2x > 8$ by graphing, and then using test intervals. Graph the solution.

Graphing Steps

1. Rewrite the inequality as a quadratic equation.
2. Find the roots (x-intercepts).
3. Sketch a graph and use the visual to solve the inequality.

→ if the quadratic is > 0, find the domain where the graph is above the x-axis

→ if the quadratic is < 0, find the domain where the graph is below the x-axis

1) $x^2 + 2x > 8$

$x^2 + 2x - 8 > 0$

2) If this was a parabola, x int are??

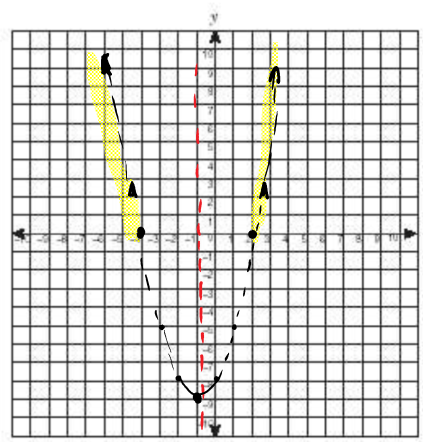
$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = 2, -4$

3) x is > 0 when

$\boxed{x < -4 \text{ \& } x > 2}$



or complete □

$y = x^2 + 2x - 8$

$y = (x^2 + 2x) - 8$

$y = (\boxed{x^2 + 2x + 1} - 1) - 8$

$y = (x+1)^2 - 9$

V (-1, 9)

ALGEBRA:

Test Interval Steps

1. Find the critical numbers (the zeros) of the inequality.
2. Make an *x*-axis diagram of the resulting test intervals.
3. Test a value from each interval using the original inequality.

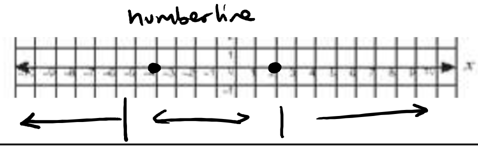
1) $x^2 + 2x > 8$

$x^2 + 2x - 8 > 0$

$(x+4)(x-2) > 0$

$-4, 2$

2) numberline



3) Test point from 3 regions

$x < -4$ pick -7	$-4 < x < 2$ pick $x = 0$	$x > 2$ pick 4
$x^2 + 2x > 8$	$x^2 + 2x > 8$	$x^2 + 2x > 8$
$(-7)^2 + 2(-7) > 8$	$0^2 + 2(0) > 8$	$4^2 + 2(4) > 8$
$49 + -14 > 8$	$0 > 8$	$16 + 8 > 8$
$35 > 8$	FALSE	$24 > 8$
TRUE REGION is GOOD	NOT A SOLUTION REGION	TRUE

SOLUTION: $x < -4$ & $x > 2$

Example – Solve $x^2 - 10x + 16 \leq 0$ using both methods and graph the solution.

*if the quadratic is ≥ 0, find the domain where the graph is **above or on** the x-axis

*if the quadratic is ≤ 0, find the domain where the graph is **below or on** the x-axis

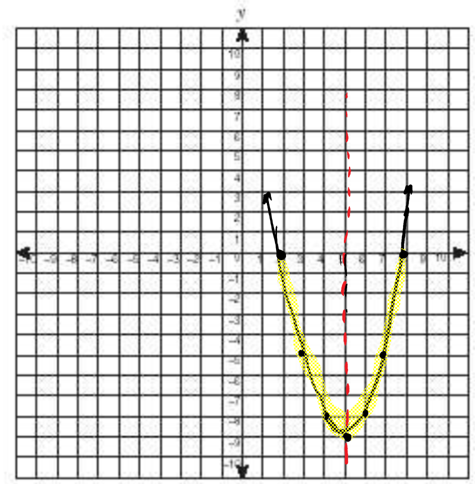
Graphing:

$x^2 - 10x + 16 \leq 0$

$(x - 2)(x - 8) \leq 0$

$x = 2, 8$

Solution $2 \leq x \leq 8$



Test Intervals:

2, 8 Test

$x \leq 2$ pick 0	$2 \leq x \leq 8$ pick 3	$x \geq 8$ pick 10
$x^2 - 10x + 16 \leq 0$	$3^2 - 10(3) + 16 \leq 0$	$10^2 - 10(10) + 16 \leq 0$
$16 \leq 0$	$9 - 30 + 16 \leq 0$	$100 - 100 + 16 \leq 0$
FALSE	$-5 \leq 0$	$16 \leq 0$
	TRUE	FALSE

Example – Graph the quadratic function $f(x) = x^2 - 6x + 9$. What is the solution to:
a) $x^2 - 6x + 9 \geq 0$ b) $x^2 - 6x + 9 > 0$ c) $x^2 - 6x + 9 \leq 0$ d) $x^2 - 6x + 9 < 0$

$x^2 - 6x + 9 = f(x)$

$(x - 3)(x - 3) = fx$

root $x = 3$

only 1 root must be the vertex

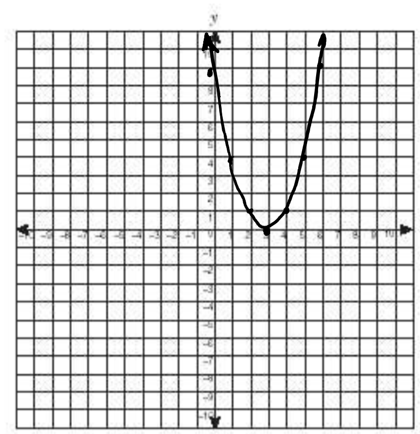
a) $x \in \mathbb{R}$

b) $x \neq 3$, $x \in \mathbb{R}$

c) $x = 3$

d) no solution

$\varnothing$



Example – Solve $-2x^2 + 7x > -12$ by graphing. Then graph the solution.

$-2x^2 + 7x + 12 > 0$ $a \cdot c = -24$ DNF

$b = 7$

ROOTS BY QUAD EQN:

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-7 \pm \sqrt{7^2 - 4(-2)(12)}}{}$

$x = \dfrac{-7 \pm \sqrt{49 + 96}}{-4}$

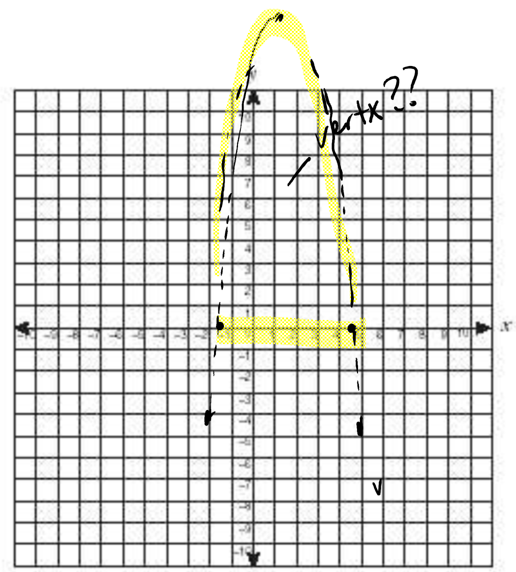
$x = \dfrac{-7 \pm \sqrt{145}}{-4}$

$x = 4.76$ ∮ -1.26

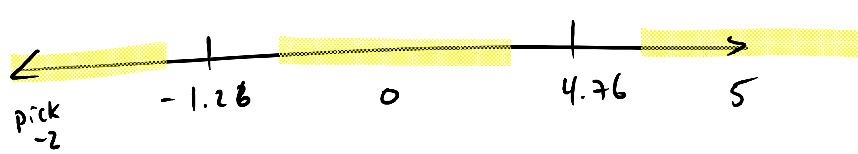
between the roots the parabola is above the axis

$\{x \mid -1.26 < x < 4.76, x \in R\}$

DO DAY 7



4.76 ∮ − 1.26



$x^2 \quad > \quad <$

9.3 – Quadratic Inequalities in Two Variables **Date:**

<u>Key Ideas:</u>

steps

Example – Solve the inequality by graphing $y + 2 < (x - 4)^2$.

1. Rearrange the inequality so y is all by itself on one side.

2. Decide whether to use a solid curve or dotted curve:

- If the inequality is ≤ or ≥, points on the parabola are included in the solution (due to the 'equals to' under the sign), so we keep the curve solid.
- If the inequality is < or >, points on the parabola are not included in the inequality, so we draw a dotted curve.

3. Graph the parabola using vertex form. The line is called the **boundary**.

4. For y > $ax^2 + bx + c$ or y ≥ $ax^2 + bx + c$, solutions to the inequality are all of the points **above** the parabola, so shade above. For y < $ax^2 + bx + c$ or y ≤ $ax^2 + bx + c$, shade **below** the parabola. The shading represents the **solution region**: all of the points that satisfy the inequality.

5. CHECK: Pick a **test point** in the shaded region and test its coordinates in the inequality. If it satisfies the inequality, you've been successful. If it doesn't satisfy the inequality, you either shaded the incorrect region, or the boundary curve has been graphed incorrectly.

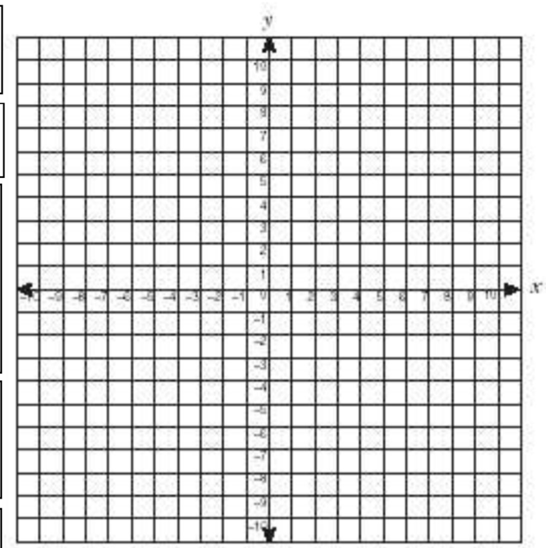
1)

2)

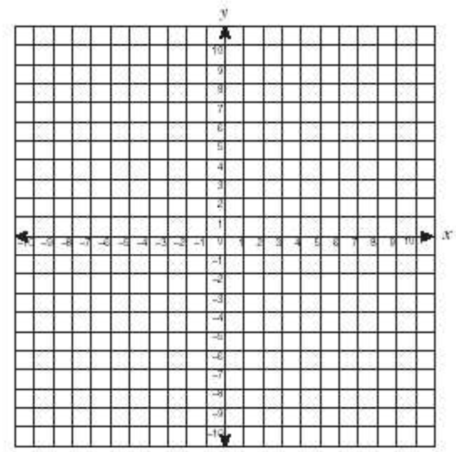
3)

4)

5)



Example – Solve by graphing: $y \leq -x^2 + 2x + 4$. Is (-1, 1) a solution? Is (2, 5)?



Example – Solve by graphing: $y + 5 \geq x^2 - 4x$.