Day 3 Algebraic solutions to Systems

January 9, 2018 2:58 PM

8.1 – Solving Systems of Equations Graphically **Date:**

Key Ideas:

- Solve with algebra to improve speed & accuracy.

Linear-Quadratic

A Linear-Quadratic System of Equations is a linear equation and a quadratic equation involving the same two variables. The solution(s) to this system are the point(s) where the line intersects the parabola (if it does at all).

Draw pictures to represent the possible number of solutions that a linear-quadratic system can have:

Quadratic-Quadratic

A Quadratic-Quadratic System of Equations is two quadratic equations involving the same variables. The solution(s) to this system are the point(s) where the parabola intersects the other parabola (if it does at all).

Draw pictures to represent the possible number of solutions that a quadratic-quadratic system can have:

Read over example 1 on p.427 of the text. Then try the 'Your Turn' at the bottom of the page. Answer in the space below:

a)	b)

Linear-Quadratic

Example – Solve the following system of equations graphically:

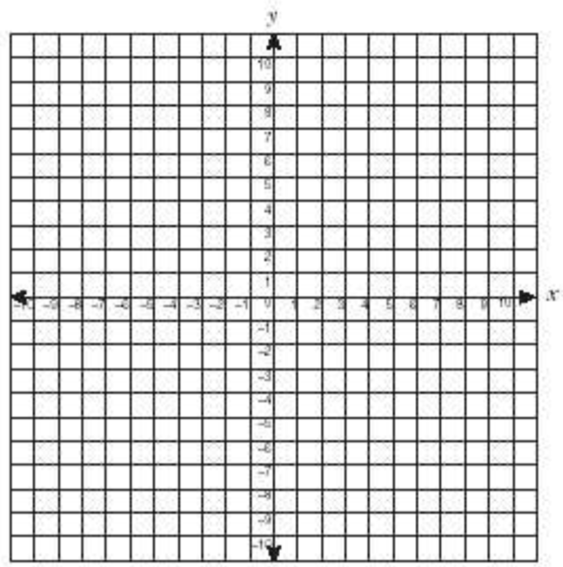
1) $4x - y + 3 = 0$
2) $2x^2 + 8x - y + 3 = 0$

a) Get the linear equation into $y = mx + b$ form and graph.
b) Complete the square and graph the quadratic equation.
c) Identify and write down the points of intersection (the solution).
d) Verify the solution by checks.

a)

b)

c)

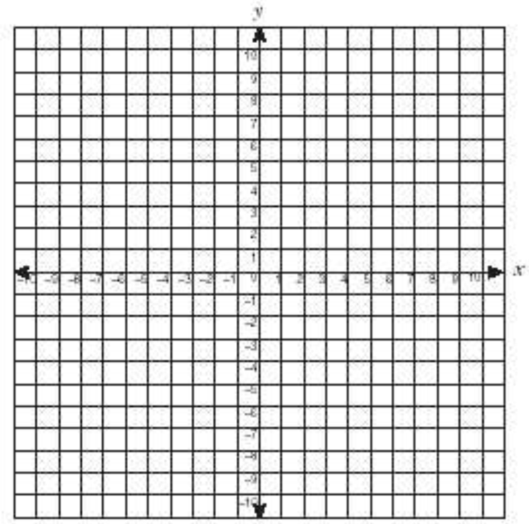


d)

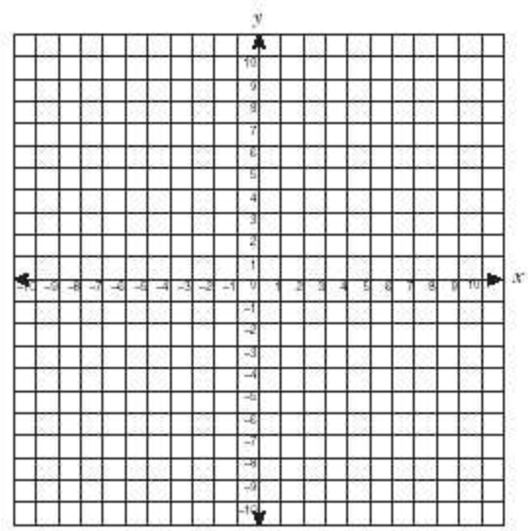
Example – Is (5, 7) a solution to the system 1) $3x^2 - 10y = 5$ and 2) $-y = x - 11$?

Quadratic-Quadratic

Example – Solve 1) $2x^2 - 8x + 7 - y = 0$ and 2) $y + x^2 - 4x + 2 = 0$



Example – Solve the system $y - x^2 + 4 = 0$ and $-2y + 2x^2 - 8 = 0$



8.2A – Solving Systems of Equations Algebraically **Date:**

Key Ideas:

Algebra is faster & more accurate than graphing.

Linear-Quadratic

For a Linear-Quadratic System of Equations, what are all the possible # of solutions?

Solutions can be found graphically, as in Section 8.1, or algebraically, using either substitution or elimination.

substitution

Example – Solve the following linear-quadratic system using **substitution**:

1) $3x + y = -9$ — Linear
2) $4x^2 - x + y = -9$ Quadratic

a) Solve the linear equation for y.
b) Substitute the linear equation for y in the quadratic equation.
c) Solve the quadratic equation by factoring (if you cannot factor, use the quadratic formula).
d) Substitute the resulting x value(s) into the original linear equation to determine the corresponding y values.

a) $3x + y = -9$

$y = -3x - 9$

b &c) $4x^2 - x + y = -9$

$4x^2 - x + (-3x - 9) = -9$

$4x^2 - 4x - 9 = -9$

$4x^2 - 4x = 0$

$4x(x - 1) = 0$

$x = 1, 0$

d) $3x + y = -9$

Find the y's

$3(1) + y = -9$

$3 + y = -9$

$y = -12$

$(1, -12)$

$3(0) + y = -9$

$y = -9$

$(0, -9)$

Example – Solve by substitution: 1) $y = x^2 - 3x - 4$ and 2) $2x - y = 4$

$2x - 4 = x^2 - 3x - 4$

$0 = x^2 - 5x$

$0 = x(x - 5)$

$x = 0, 5$

$(0, -4)$ & $(5, 6)$

$2x - 4 = y$

$x = 0$: $2(0) - 4 = y$, $-4 = y$

$x = 5$: $2(5) - 4 = y$, $10 - 4 = y$, $6 = y$

elimination

Example – Solve the following linear-quadratic system using **elimination**:

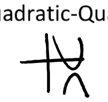
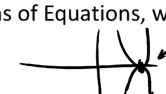
1) $5x - y = 10$ — linear
2) $x^2 + x - 2y = 0$ — quadratic

a) Align the terms with the same degree. Since the squared term the variable x, eliminate the y-term.
b) Multiply one or more of the equations if necessary to have the same coefficient for y.
c) Add or subtract the two equations to eliminate y.
d) Solve the resulting quadratic equation by factoring or the quadratic formula to find the x coordinates of the solution(s).
e) Substitute the resulting x value(s) into the original linear equation to determine the corresponding y values.

a, b, c)	d) / e)
$x^2 + x - 2y = 0$ $(5x - y = 10) \times -2$ $x^2 + x - 2y = 0$ ADD $-10x + 2y = -20$ $x^2 - 9x = -20$ $x^2 - 9x + 20 = 0$ $(x-4)(x-5) = 0$ $x = 4, 5$	d) $x^2 - 9x + 20 = 0$ $(x-4)(x-5) = 0$ $x = 4, 5$ e) Get "y's" $5x - y = 10$ $y = 10$ $y = 15$ $(4, 10)$ $(5, 15)$

Quadratic-Quadratic

For a Quadratic-Quadratic Systems of Equations, what are all the possible # of solutions?

 0 solution  1 sol  2 sol

Example – Solve the following system first by substitution, then by elimination.

1) $6x^2 - x - y = -1$
2) $4x^2 - 4x - y = -6$

Solve both for y

Substitution $6x^2 - x + 1 = y$ & $4x^2 - 4x + 6 = y$

$$6x^2 - x + 1 = 4x^2 - 4x + 6$$

$$2x^2 + 3x - 5 = 0$$

$a \times c = -10$
$b = 3$
$5, -2$

$2x^2 + 5x \mid -2x - 5 = 0$
$x(2x+5) \mid -1(2x+5) = 0$

$(x-1)(2x+5) = 0$

$x = 1, -5/2$

$(1, 6)$

$6x^2 - x - y = -1$
$6\left(\frac{-5}{2}\right)^2 - \left(\frac{-5}{2}\right) - y = -1$

$$3\cancel{6}\left(\frac{25}{\cancel{4}_{2}}\right)+\frac{5}{2}-y=-1$$

$$\frac{75}{2}+\frac{5}{2}-y=-1$$

$$\frac{80}{2}-y=-1$$

$$40-y=-1$$

$$-y=-41$$

$$y=41$$

1) $6x^2 - x - y = -1$
2) $4x^2 - 4x - y = -6$

Elimination:

$$\left(-5/2, 41\right)$$

$$6x^2 - x - y = -1$$

Sub $\quad 4x^2 - 4x - y = -6$

$$2x^2 + 3x = 5$$

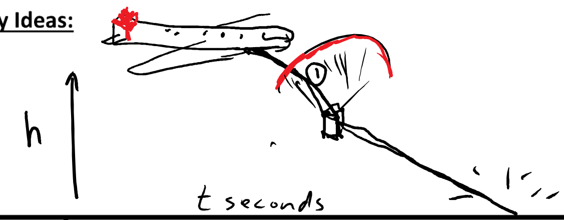
$$\boxed{2x^2 + 3x - 5 = 0}$$

see above

8.2B – Word Problems Date:

Key Ideas:

Key Ideas:



Example – A Canadian cargo plane drops a crate of emergency supplies to aid-workers on the ground. The crate drops freely at first before a parachute opens to bring the crate gently to the ground. The crate's height, h, in metres, above the ground t seconds after leaving the aircraft is given by the following two equations. $h = -4.9t^2 + 900$ represents the height of the crate during freefall. $h = -4t + 500$ represents the height of the crate with the parachute open. Find intersection point of the eqns.

a) How long after the crate leaves the aircraft ~~does the~~ parachute open? Express your answer to the nearest hundredth of a second. 9.45 s

b) What height above the ground is the crate ~~when the~~ parachute opens? Express your answer to the nearest metre.

c) Verify your solution.

$-4.9t^2 + 900 = -4t + 500$

$\boxed{0 = 4.9t^2 - 4t - 400}$

$t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{4 \pm \sqrt{(-4)^2 - 4(4.9)(-400)}}{2(4.9)}$

$= \dfrac{4 \pm \sqrt{7856}}{9.8}$

$\dfrac{4 \pm 88.634}{9.8}$

$+ \rightarrow 9.452$

$- \rightarrow$ REJECT b/c $(-)$

$h = -4t + 500$

$h = -4(9.45) + 500$

$h = 462$ m

12 & 17   -15, 31

\+ or −
but not
fractions
decimals

Example – A certain website offers online interactive puzzles, but the puzzle-makers present the following problem for entry to their site. "Determine two integers such that the sum of the smaller number and twice the larger number is 46. Also, when the square of the smaller number is decreased by three times the larger, the result is 93. By determining the smaller and larger numbers, use it as a password to gain access to the site.

Let X = smaller #
Let Y = larger #

① $X + 2Y = 46$ ② $X^2 - 3Y = 93$

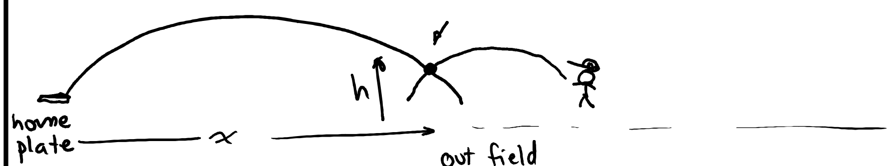
$X = 46 - 2Y$

$$(46-2Y)^2 - 3Y = 93$$
$$2116 - 184y + 4y^2 - 3y = 93$$
$$4y^2 - 187y + 2023 = 0$$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$
$$+187 \pm \sqrt{(-187)^2 - 4(4)(2023)}$$
$$\frac{187 \pm 51}{8}$$

Example – Mike hits the baseball and it travels on a path modeled by $h = -0.1x^2 + 2x$. John is in the outfield directly in line with the path of the ball. He runs toward the ball and jumps to try to catch it. His jump is modeled by the equation $h = -x^2 + 39x - 378$. In both equations, x is the horizontal distance in metres from home plate and h is the height of the ball above the ground in metres.

a) Solve the system algebraically. Round your answer to the nearest hundredth.
b) Explain the meaning of the point of intersection. What assumptions are you making?



$$0.1X^2 + 2X = -x^2 + 39x - 378$$
$$0.9X^2 - 37x + 378 = 0$$

quad eqn ↓

X = 22.5   18.96
sub in   sub in
h = -4.77   1.97 m
Reject