GraphingStudentnotes

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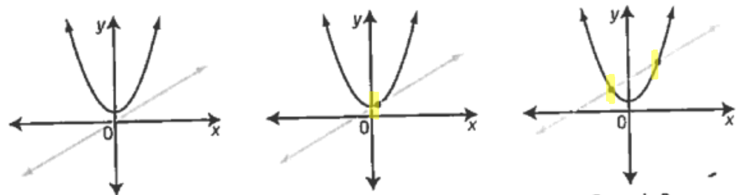
Chapter 8 Systems of Equations

8.1 Solving Systems of Equations Graphically

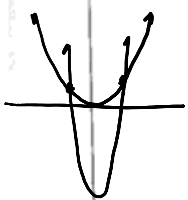
KEY IDEAS

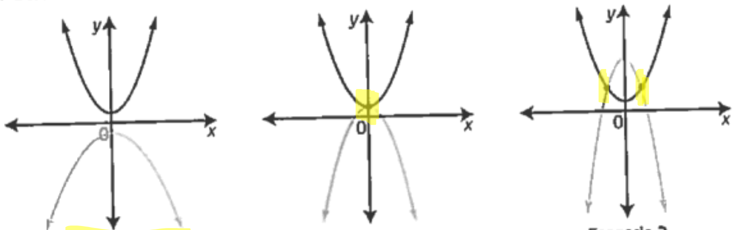
- A system of equations is two or more different equations involving the same variabes.
- Determining the solution to a system of equations means determining point(s) that are common to both equations. A common point is called a point of intersection.
- Graphing a linear equation and a quadratic equation can produce one of three possible scenarios:



Scenario 1 | Scenario 2 | Scenario 3

  - In Scenario 1, the line and parabola do not intersect. There is no solution.
  - In Scenario 2, the line and parabola intersect at one point. There is one solution.
  - In Scenario 3, the line and parabola intersect at two different points. There are two solutions.
- Graphing two quadratic equations can produce one of four possible scenarios. In the scenario in which the quadratic equations are the same, there are an infinite number of solutions. The other three scenarios are illustrated:





Scenario 1 | Scenario 2 | Scenario 3

  - In Scenario 1, the parabolas do not intersect. There is no solution.
  - In Scenario 2, the parabolas intersect at one point. There is one solution.
  - In Scenario 3, the parabolas intersect at two different points. There are two solutions.
- Once you have determined the point of intersection, remember to check the ordered pair in both of the original equations.

Working Example 1: Solve a System of Linear-Quadratic Equations Graphically

a) Solve the following system of equations graphically.

$y + 3x^2 - 2x - 4 = 0$ – QUADRATIC : PARABOLA
$y + x + 2 = 0$ LINEAR straight line

b) Verify the solution.

Solution

a) Isolate y in the first equation. Perform the opposite operations to both sides of the equation to isolate y.

$y + 3x^2 - 2x - 4 = 0$
$y = -3x^2 + 2x + 4$

Why do you have to perform inverse operations to both sides of the equation?

New equation: $y = -3x^2 + 2x + 4$

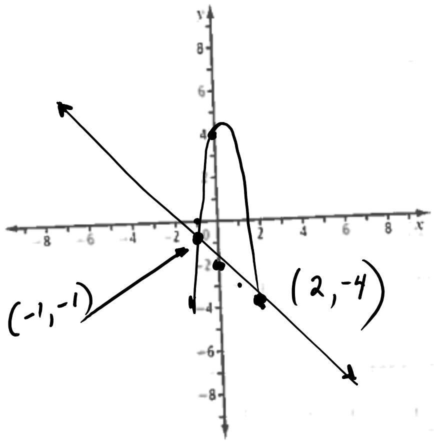
Isolate y in the second equation.

$y + x + 2 = 0$
$y = -x - 2$

New equation: $y = -x - 2$

Enter the equations into your graphing calculator. Use the intersect feature to determine the point(s) of intersection.

Sketch the graph of each equation and label the point(s) of intersection.



How many points of intersection are there? How many solutions are there?

$(-1, -1)$

b) Verify your solution by substituting the first solution into both of the original equations.

Why do you have to use the original equations to verify your solution(s)?

PARABOLA

Left Side	Right Side
$y + 3x^2 - 2x - 4$	0
$(-1) + 3(-1)^2 - 2(-1) - 4$	
$-1 + 3 + 2 - 4$	
0	$= 0$ ✓

Left Side	Right Side
$y + x + 2$	0
$(-1) + (-1) + 2$	
0	$= 0$ ✓

Verified Solution

Verify your solution by substituting the second solution into both of the original equations.

$(2, -4)$

Left Side	Right Side
$y + 3x^2 - 2x - 4$	0
$-4 + 3(2)^2 - 2(2) - 4$	0
$-4 + 12 - 4 - 4$	$= 0$
0	$= 0$ ✓

Left Side	Right Side
$y + x + 2$	0
$-4 + 2 + 2$	
0	$= 0$ ✓

Verified solution

Working Example 2: Solve a System of Quadratic-Quadratic Equations Graphically

a) Solve the following system of equations graphically.

① $y - 3x^2 + 12x = 16$ → quadratic – parabola

② $y + 4x^2 - 16x = -12$ → quadratic – parabola

b) Verify the solution.

Solution

a) Isolate y in the first equation.

$y - 3x^2 + 12x = 16$

$y - 3x^2 + 12x = 16$

New equation: $y = 3x^2 - 12x + 16$

up

Isolate y in the second equation.

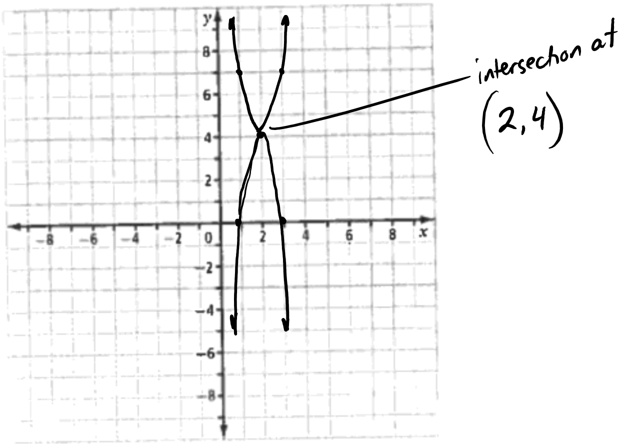
$y + 4x^2 - 16x = -12$

New equation: $y = -4x^2 + 16x - 12$

down

Enter the equations into your graphing calculator. Use the intersect feature to determine the point(s) of intersection.

Sketch the graph of each equation and label the point(s) of intersection.



How many points of intersection are there? How many solutions are there?

x y
(2,4)

b) Verify your solution by substituting the solution into both of the original equations.

Left Side	Right Side
$y - 3x^2 + 12x$	16
$4 - 3(2)^2 + 12(2)$	
$4 - 12 + 24$	
$-8 + 24$	
16	16 ✓

Left Side	Right Side
$y + 4x^2 - 16x$	−12
$4 + 4(2)^2 - 16(2)$	
$4 + 16 - 32$	−12
$20 - 32$	
−12	−12 ✓