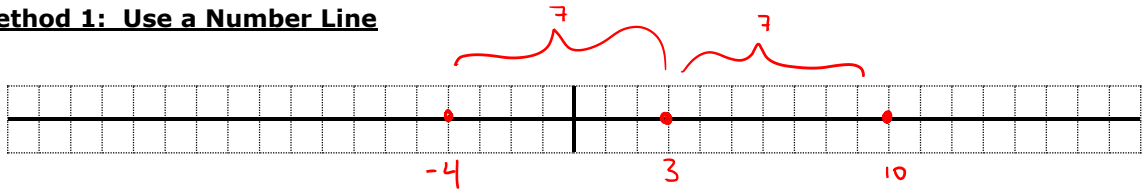
SOLVE AN ABSOLUTE VALUE EQUATION

EX. 1 Solve $|x-3|=7$. Explain your steps for each method.

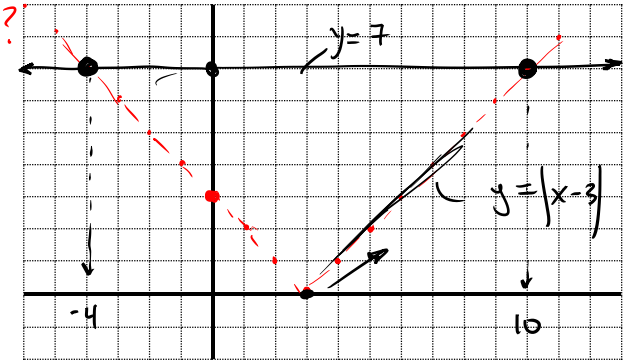
Method 1: Use a Number Line



Method 2: Use a Graph

Graph each side → intersect?

$|x-3|=y$ $\quad y=7$



Method 3: Use Algebra

① Isolate the absolute value to one side

② Use absolute value definition to consider the two cases (including restrictions on domain or conditions on variable)
Case 1: positive or zero
Case 2: negative

③ Solve each equation from step ② for variable.

④ Check for extraneous roots and reject them.

$|x-3|=7$

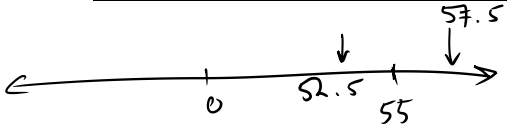
+ case	− case
$x \geq 3$	$x < 3$
$x-3=7$	$-(x-3)=7$
$x=10$	$-x+3=7$
	$-x=4$
	$x=-4$

SOLVE AN ABSOLUTE VALUE PROBLEM

EX. 2 A computer programs sets the ideal mass before baking at 55 g but allows a tolerance of ± 2.5 g, solve an absolute value equation for the maximum and minimum mass, m of batter for cookies at this factory.

$|m-55g|=2.5$

Method 1: Use a Number Line



two masses are 52.5 minimum & 57.5 max

Method 2: Use Algebra

+ case	− case
$m \geq 55$	$m < 55$
$m-55=2.5$	$-(m-55)=2.5$
$m=2.5+55$	$-m+55=2.5$
$m=57.5g$	$-m=2.5-55$
"max"	$-m=-52.5$
	$m=52.5g$
	MIN

ABSOLUTE VALUE EQUATION WITH AN EXTRANEOUS ROOT

isolated ☺ $2x-5=0$, $2x=5$, $x=\frac{5}{2}$

EX. 3 Use the definition of absolute value and algebra to solve $|2x-5|=5-3x$

+ case $x \geq 5/2$

$2x-5=5-3x$

$5x-5=5$

$5x=10$

$x=2$ REJECT EXTRANEOUS

− case $x < 5/2$

$-(2x-5)=5-3x$

$-2x+5=5-3x$

$x+5=5$

$x=0$ ✓

ABSOLUTE VALUE EQUATION WITH NO SOLUTION

EX. 4 Solve $|3x-4|+12=9$ isolate abs value

$|3x-4|=9-12$

$|3x-4|=-3$

no solution − empty set

ABSOLUTE VALUE EQUATION INVOLVING A QUADRATIC EXPRESSION

EX. 5 Solve $|x^2-2x|=1$. wee bit trickier

+ case $x \leq 0$ or $x \geq 2$

$x^2-2x=1$

$x^2-2x-1=0$ DNF

$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\frac{2 \pm \sqrt{4+4}}{2}$

$\frac{2 \pm \sqrt{8}}{2} \rightarrow \frac{2 \pm 2\sqrt{2}}{2} \rightarrow 1 \pm \sqrt{2}$

$+2.4$

-0.4

− case $0 < x < 2$

$-(x^2-2x)=1$

$-x^2+2x=1$

$0=x^2-2x+1$

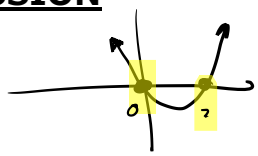
$0=(x-1)(x-1)$

soln $x=1$

$x^2-2x=0$

$x(x-2)=0$

$x=0, 2$



ABSOLUTE VALUE EQUATION INVOLVING LINEAR & QUADRATIC EXPRESSION

$|x-10|$ $\begin{cases} x-10 & x \geq 10 \text{ + case} \\ -(x-10) & x < 10 \text{ − case} \end{cases}$

EX. 6 Solve $|x-10|=x^2-10x$

+ case $x \geq 10$

$x-10=x^2-10x$

$0=x^2-11x+10$

$0=(x-1)(x-10)$

Sol $x=1, 10$ ✓ REJECT

− case $x < 10$

$-(x-10)=x^2-10x$

$-x+10=x^2-10x$

$0=x^2-9x-10$

$0=(x-10)(x+1)$

$x=10, -1$ reject, keep

10, −1